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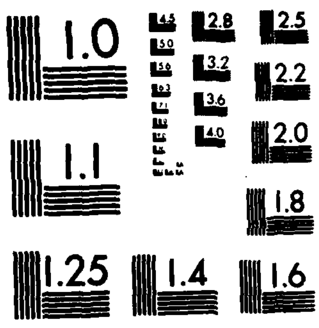

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A NEW GENERALIZED CROSS-CORRELATOR

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ABSTRACT

A new Generalized Cross-Correlator (G.C.C.) for the passive time delay estimation problem is presented. The interpretation of this G.C.C. is that of estimating the cross-correlation function by cross-correlating the least mean square estimates of the signal component in each of the observed waveforms. The implementation is simply a G.C.C. with the weighting filter equal to the magnitude coherency squared. Numerical evaluation of the performance of this processor and a robust version indicates that they compare favorably to some of the well known G.C.C. procedures.

INTRODUCTION

The estimation of propagation delay in a common signal arriving at two spatially separated sensors is a problem which has received much attention in the literature. Cross-correlation methods are particularly popular because of the richness and variety of processors within this class and the general ease of implementation. Of these, the *Hannan Thompson* ($H.T.$) [1,5], the *Eckart* [1] and the *Hassab-Boucher* ($H.B.$) [2] are examples of "optimum" processors which maximize some performance criteria. On the other hand, the *SCOT* [7] and the simple cross-correlator ($C.C.$) are examples of "Ad Hoc" or intuitive correlation type processors. In general the optimum processors are very sensitive to deviations from the assumed signal and noise characteristics. By way of contrast, the $C.C.$ and $SCOT$ appear to be more robust to these deviations from the nominal. However, these later processors can have very poor performance at the nominal point.

In the following development an intuitive approach to the problem yields another type of (generalized) cross-correlator. This is the ***Wiener Processor***, or $W.P.$, which has a simple form. Yet preliminary results indicate it outperforms most of the other above named processors when compared under various performance criteria for the few important cases considered in this paper. A "robust" version of the $W.P.$ also indicates potentially good performance relative to the others under uncertain spectra.

I. PROBLEM STATEMENT AND BACKGROUND

We first consider a system model generating the observations in Fig. I. We observe Gaussian, ergodic, wide-sense stationary processes $x_1(t)$ and $x_2(t)$ over a time interval [0,T] which contain uncorrelated noises $n_1(t)$ and $n_2(t)$ and signals $s(t)$ and $s_o(t)$ respectively. We assume $c(t)$ is a linear time invariant channel having a transfer function $C(\omega)$ with unknown linear phase so that $s_o(t)$ is a delayed but possibly distorted version of $s(t)$. Furthermore, we assume that the noises are uncorrelated with the signal and that T is much greater than the correlation time of $x_1(t)$ and $x_2(t)$. The object is then to estimate the time delay, D, asociated with the channel.

We define the sample cross-correlation:

$$\hat{R}_{12}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{G}_{12}(\omega)e^{j\omega\tau}d\omega \quad . \qquad (1.1)$$

Here $\hat{G}_{12}(\omega)$ is some unbiased, consistent estimate of the cross-spectrum $G_{12}(\omega)$. For definiteness we assume that $\hat{G}_{12}(\omega)$ is obtained by the Bartlett Procedure [19] consisting of segment averaging periodogram type estimates $\frac{m}{T}X_{1k}^*(\omega)X_{2k}(\omega)$ where m is the total number of subdivisions of [0,T] and $X_{1k}(\omega)$ and $X_{2k}(\omega)$ are the Fourier Transforms of $x_1(t)$ and $x_2(t)$ on the k-th segment.

For very large observation time and suitable m, $\hat{R}_{12}(\tau)$ gives a good approximation to the true cross-correlation function which has a global peak at D. In fact if $c(t)$ is pure delay and $s(t)$ is white, the cross-correlation function is a delta function at the true delay. For finite

observation time we can decompose $\hat{R}_{12}(\tau)$ into the sum of four terms:

$$\hat{R}_{12}(\tau) = c(\tau) * \hat{R}_{ss}(\tau) + c(\tau) * \hat{R}_{n_1 s}(\tau) + \hat{R}_{sn_2}(\tau) + \hat{R}_{n_1 n_2}(\tau) \qquad (1.2)$$

where "*" denotes convolution. Here $\hat{R}_{ss}(\tau)$ is an estimate of the signal autocorrelation function $R_{ss}(\tau)$ and $\hat{R}_{n_1 s}(\tau)$, $\hat{R}_{sn_2}(\tau)$ and $\hat{R}_{n_1 n_2}(\tau)$ are estimates of the cross-correlation between the signal and noise terms in the observations. In the limit the sample cross-correlation converges to $c(\tau) * R_{ss}(\tau)$ which displays an absolute maximum at D. Thus, it is the last three terms in Eqn. (1.2) which constitute zero mean disturbances affecting peak resolution of the first term. This suggests prefiltering the sample cross-correlation with a filter $W(\omega)$ to obtain better resolution of the peak at D, where $W(\omega)$ has zero phase. This scheme is referred to as the generalized cross-correlation method or the Generalized Cross-Correlator (G.C.C.) and is illustrated in Fig. II (The content of the dashed box is only symbolic for the operation in Eqn (1.1)). We denote the G.C.C. output waveform $R^{g}_{12}(\tau)$. Therefore we have:

$$R^{g}_{12}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{G}_{12}(\omega) W(\omega) e^{j\omega\tau} d\omega \ . \qquad (1.3)$$

When $W(\omega)$ is unity the resulting G.C.C. is called the simple cross-correlator ($C.C.$). Considering the first term in Eqn. (1.2) as a "signal" in additive noise, classical optimal filtering theory can be applied to derive filters $W(\omega)$ which maximize signal-to-noise ratio.

Letting the last three terms of Eqn (1.2) be characterized as "noise" we can define a signal-to-noise ratio as the energy at the output due to

the global peak of $c(\tau) * \hat{R}_{ss}(\tau)$ divided by the output power due to the three noise terms. We will denote this by SNR_1. For the specific cross-spectral estimation procedure outlined above, the noise power has been derived and is given by:

$$\sigma_n^2 = \frac{k}{2\pi} \int_{-\infty}^{\infty} G_{11}(\omega) G_{22}(\omega)(1-\gamma_{12}^2(\omega)) |W(\omega)|^2 d\omega \tag{1.4}$$

k is a positive constant inversely proportional to T, the observation time. $G_{11}(\omega)$ and $G_{22}(\omega)$ are the power spectral densities of the observations $x_1(t)$ and $x_2(t)$ respectively. $\gamma_{12}^2(\omega)$ is the magnitude coherency squared, where, defining the true cross-spectrum $G_{12}(\omega)$,

$$\gamma_{12}^2(\omega) = \frac{|G_{12}(\omega)|^2}{G_{11}(\omega) G_{22}(\omega)} . \tag{1.5}$$

Then from the defining relation:

$$SNR_1 = \frac{[E\{R_{12}^g(\tau)|_{\tau=D}\}]^2}{\sigma_n^2}$$

we obtain:

$$SNR_1 = \frac{[\frac{1}{2\pi} \int_{-\infty}^{\infty} |G_{12}(\omega)| W(\omega) d\omega]^2}{\frac{k}{2\pi} \int_{-\infty}^{\infty} G_{11}(\omega) G_{22}(\omega)(1-\gamma_{12}^2(\omega)) |W(\omega)|^2 d\omega} \tag{1.6}$$

The maximum is obtained through the Schwarz inequality and yields the $H.T.$ processor. The same result is derived in [5] as the result of minimizing the local variance of the delay estimate over the entire G.C.C. class,

and in [1] as the result of maximum likelihood estimation. The filter is:

$$W_{H.T.}(\omega) = \frac{1}{|G_{12}(\omega)|}\frac{\gamma_{12}^2(\omega)}{1-\gamma_{12}^2(\omega)} . \tag{1.7}$$

Neglecting the effect of the signal and noise cross terms , $c(\tau)*\hat{R}_{sn_1}(\tau)$ and $\hat{R}_{sn_2}(\tau)$ in Eqn. (1.2), gives another characterization of the noise in the cross-correlation domain. With this definition of noise another signal-to-noise ratio is defined in [8], SNR_2, which is shown to be maximized by the *Eckart* processor $W_{EK}(\omega)$:

$$SNR_2 = \frac{[\frac{1}{2\pi}\int_{-\infty}^{\infty}|G_{12}(\omega)|W(\omega)d\omega]^2}{\frac{k}{2\pi}\int_{-\infty}^{\infty}G_{n_1}(\omega)G_{n_2}(\omega)|W(\omega)|^2d\omega} \tag{1.8}$$

$$W_{EK}(\omega) = \frac{|G_{12}(\omega)|}{G_{n_1}(\omega)G_{n_2}(\omega)} \tag{1.9}$$

Where $G_{n_1}(\omega)$ and $G_{n_2}(\omega)$ are the auto-spectra of the noises $n_1(t)$ and $n_2(t)$ respectively. Note in terms of the spectra of the observables $x_1(t)$ and $x_2(t)$ the filter takes the form

$$W_{EK}(\omega) = \frac{|G_{12}(\omega)|}{(G_{11}(\omega)-|G_{12}(\omega)|)(G_{22}(\omega)-|G_{12}(\omega)|)}$$

which attests to the complexity of implementing the *Eckart* in general.

Hassab and Boucher [2] take the approach of maximizing a signal-to-noise ratio, SNR_3, defined as the ratio of the expected peak energy at the true delay to the total statistical variation of the output of the G.C.C. This, in some sense, lumps the "signal", $c(\tau)*\hat{R}_{ss}(\tau)$, variation into the

noise terms and yields the *H.B.* filter $W_{H.B.}(\omega)$:

$$SNR_3 = \frac{\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} |G_{12}(\omega)| W(\omega) d\omega\right]^2}{\frac{k}{2\pi}\int_{-\infty}^{\infty} G_{11}(\omega) G_{22}(\omega) |W(\omega)|^2 d\omega} \tag{1.10}$$

$$W_{H.B.}(\omega) = \frac{|G_{12}(\omega)|}{G_{11}(\omega) G_{22}(\omega)} . \tag{1.11}$$

The *H.B.* is similar to the *SCOT* introduced by Carter *et al* [7] in that it suppresses the cross-spectral estimate in ω-regions of both high and low signal-to-noise ratio in an attempt to reject strong tonals in the observations.

II. THE WIENER PROCESSOR

Here a slightly different approach is taken to derive an optimal filter. We deal directly with the quantities in the observation time domain (i.e. Fig. I). The procedure is motivated by the following argument. If we knew the signal $s(t)$ and the filtered version $s_o(t)$ exactly then from the linearity of the phase of the channel the time delay could be estimated exactly by detecting the peak of the sample cross-correlation of $s(t)$ and $s_o(t)$. Therefore we simply try to estimate the signal $s(t)$ as best we can from the observations $x_1(t)$ and the channel output signal $s_o(t)$ from $x_2(t)$ by minimizing the mean square errors:

$$E\{(s(t) - \hat{s}(t))^2\} = \min \tag{2.1}$$

$$E\{(s_o(t) - \hat{s}_o(t))^2\} = \min \tag{2.2}$$

where

$$\hat{s}(t) = \int_{-T}^{T} x_1(\sigma) h_1(t-\sigma) d\sigma$$

$$\hat{s}_o(t) = \int_{-T}^{T} x_2(\sigma) h_2(t-\sigma) d\sigma \ .$$

The above procedure is illustrated in Fig. III. Given the channel characteristic $C(\omega)$ the solutions to Eqn. (2.1) and (2.2) are the Wiener filters $H_1(\omega)$ and $H_2(\omega)$:

$$H_1(\omega) = \frac{G_{ss}(\omega)}{G_{ss}(\omega) + G_{n_1 n_1}(\omega)}$$

$$H_2(\omega) = \frac{G_{ss}(\omega)|C(\omega)|^2}{G_{ss}(\omega)|C(\omega)|^2 + G_{n_2 n_2}(\omega)} \ .$$

Noting that $G_{12}(\omega) = C(\omega) G_{ss}(\omega)$ we can express the above filters in terms

of the quantities derived from the observables:

$$H_1(\omega) = \frac{1}{C(\omega)} \frac{G_{12}(\omega)}{G_{11}(\omega)} \tag{2.3}$$

$$H_2(\omega) = C^*(\omega) \frac{G_{12}(\omega)}{G_{22}(\omega)} . \tag{2.4}$$

where $C^*(\omega)$ is the complex conjugate of $C(\omega)$.

With these filters the sample cross-correlation of the least mean square error estimates of $s(t)$ and $s_o(t)$ yields the estimate of the cross correlation function:

$$\mathbf{R}_{12}^{WP}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{T} \sum_{k=1}^{m} \hat{S}_k^*(\omega) \hat{S}_{o_k}(\omega) e^{j\omega\tau} d\omega \tag{2.5}$$

where

$$\hat{S}_k(\omega) = H_1(\omega) X_{1k}(\omega) \tag{2.6}$$

$$\hat{S}_{o_k}(\omega) = H_2(\omega) X_{2k}(\omega) \tag{2.7}$$

$X_{1k}(\omega)$ and $X_{2k}(\omega)$ are, as before, the finite time Fourier transforms of $x_1(t)$ and $x_2(t)$ on the k-th segment of [0,T]. Regrouping terms in (2.5) we obtain

$$\mathbf{R}_{12}^{WP}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{G}_{12}(\omega) \frac{|G_{12}(\omega)|^2}{G_{11}(\omega) G_{22}(\omega)} e^{j\omega\tau} d\omega \tag{2.8}$$

Comparing Eqn. (2.8) with Eqn. (1.3) we have the result that the *W.P.* is equivalent to using a Generalized Cross-Correlator with the filter $W(\omega)$ equal to the magnitude coherency squared.

It should be emphasized that even though the Wiener filters H_1 and H_2 involve the knowledge of the channel $C(\omega)$ itself the G.C.C. equivalent

processor does not impose this requirement. In fact, as far as the cross-correlation estimate of time delay is concerned, the actual channel is immaterial to the peak detection procedure in the cross-correlation domain. Hence the Wiener filter implementation (Fig. III) with $C(\omega)$ arbitrarily set to unity in Eqns. (2.3) and (2.4) is equivalent to any other choice of $C(\omega)$ for the time delay estimation problem.

The definition of "additive noise" leading to the signal-to-noise ratio SNR_1, Eqn. (1.4), yields the interpretation that $\gamma_{12}^2(\omega)$ is a measure of the proportion of the power spectral density of the sample cross-correlation, $\hat{\mathbf{R}}_{12}(\tau)$, which is due to the sample autocorrelation $\hat{\mathbf{R}}_{ss}(\tau)$ (the first term in Eqn. (1.2)). Thus the *W.P.* de-emphasizes those ω-regions where the sample cross-spectrum is most likely to be a highly inaccurate estimate of the true cross-spectrum. This is not surprising given the ***raison d'être*** of the *W.P.* which is to accurately estimate the smoothed sample autocorrelation, $\hat{\mathbf{R}}_{ss}(\tau)*c(\tau)$.

The *W.P.* does not of course maximize the signal-to-noise ratio in general. If we examine the optimal processor for SNR_1, the *H.T.* (Eqn. (1.7)), we see that it has the additional ability to overemphasize as well as to de-emphasize the cross-spectral estimate according to the function $\gamma_{12}^2(\omega)/(1-\gamma_{12}^2(\omega))$. (Actually in [5] the above function is shown to be inversely proportional to the variance of the phase estimate $\hat{G}_{12}(\omega)/|\hat{G}_{12}(\omega)|$ with respect to the true phase of the cross-spectrum). However, in situations where the coherence is low, and the signal spectrum nearly flat, the *H.T.* and the *W.P.* are virtually identical and exhibit identical performance (Eqn. (1.7) becomes proportional to γ_{12}^2).

It is also observed that the *W.P.* is equivalent to the *H.B.* for nearly flat signal spectra and also to the *Eckart* if we add a low signal-to-noise ratio condition:

$$W_{H.B.}(\omega) = \frac{1}{|G_{12}(\omega)|}\gamma_{12}^2(\omega) = \frac{1}{|G_{12}(\omega)|}W_{W.P.}(\omega)$$

$$W_{EK}(\omega) = \frac{G_{ss}(\omega)}{G_{n_1}(\omega)G_{n_2}(\omega)} \approx \frac{1}{|G_{12}(\omega)|}\gamma_{12}^2(\omega)$$

The above signal-to-noise ratio condition is that $G_{ss}(\omega)$ be uniformly small as compared to $G_{n_1}(\omega)$ and $G_{n_2}(\omega)$.

III. PRACTICAL CONSIDERATIONS

In practice the various spectral quantities that appear in the expressions for the optimal filters are unknown (either partially or totally) so that the filters are not directly implementable. Two approaches to this problem are possible. We either estimate the spectra and substitute the estimates into the aforementioned filters (totally unknown spectra) or we search for a robust solution over a range of spectra perturbed from some nominal point (partially unknown spectra). In this section the estimation approach is briefly discussed and in the next section the robust approach is considered.

Several simulation studies have recently appeared in the literature which exercise the various filters for known signal and noise spectra. To the extent that generalizations are possible from a limited set of simulations, they have shown that the *H.T.* processor seems to outperform the others such as: the *SCOT* and simple cross-correlator [10]; the *Eckart*; and the *H.B.* for moderately broadband signals [11,12].

When the spectral quantities are totally unknown the *H.T.* is no longer implementable. Two main approaches to optimal time delay estimation have been investigated. The exact maximum likelihood procedure for unknown spectra was derived in [9] by Hannan and Thompson but it involves the simultaneous solution of several coupled equations at a large number of grid points in frequency. The other approach, the ***Hamon Hannan procedure*** *(H.H.)* [4], directly substitutes spectral estimates into the *H.T.* to obtain the G.C.C. structure:

$$R_{12}^{HH}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\hat{\gamma}_{12}^2(\omega)}{1-\hat{\gamma}_{12}^2(\omega)} e^{j(\hat{\varphi}(\omega)+\omega\tau)} d\omega \qquad (3.1)$$

where $\hat{\gamma}_{12}^2(\omega)$ is an estimate of magnitude squared coherence and $\hat{\varphi}(\omega)$ is an estimate of the phase $\varphi(\omega)$ of the cross-spectrum between the observations. This procedure converges to the *H.T.* in the limit of large observation times for suitable estimates of γ_{12}^2 and φ.

The *H.H.* procedure was simulated in [10] and for the specific spectral estimates reported, the *H.H.* (therein referred to as the *A.M.L.*) was shown to perform badly even with respect to the (*C.C.*) for lowpass and bandpass flat signal spectra. Indeed a simple local analysis of the filter estimation error for the *H.H.* filter, $W_{H.H.}(\omega) = \hat{\gamma}_{12}^2(\omega)/(1-\hat{\gamma}_{12}^2(\omega))$, yields the variance:

$$var(W_{H.H.}(\omega)) \approx \frac{1}{(1-\gamma_{12}^2(\omega))^4}\; var(\hat{\gamma}_{12}^2(\omega)) \; .$$

For the specific coherence estimate developed in [13] by Carter and used in Carter's program for time-delay estimation [14] the filter estimate has a variance:

$$var(W_{H.H.}(\omega)) \approx \frac{K_1}{T}\frac{\gamma_{12}^2(\omega)}{(1-\gamma_{12}^2(\omega))^2}$$

where K_1 is a positive constant. Therefore the *H.H.* may vastly underweight those ω-regions where coherence is highest because of the large slope of $W_{H.H.}$ as a function of the coherence estimate when $\hat{\gamma}_{12}^2$ is near unity. In addition, using averaged type coherence estimates like that in [13] $\hat{\gamma}_{12}^2(\omega)$ can exhibit a large negative bias when the phase of $G_{12}(\omega)$ is rapidly varying [4]. This additional bias term in Eqn. (3.2) could lead to a

drastic increase in the error of the filter estimate in ω-regions of high coherence. The net effect would be to decrease output signal-to-noise ratio, dramatically raising the variance of the time delay estimate.

The *W.P.* on the other hand can be implemented by directly filtering the cross-spectral estimate with the coherence estimate, the resultant filter having potentially less bias in both the low and high coherence ω-regions than the *H.H.* filter. This improved filter estimate could translate into improved time delay estimator performance in some cases. The practical performance of this implementation of the *W.P.* remains to be investigated.

IV. A ROBUST VERSION OF THE WIENER PROCESSOR

When the spectra underlying the observations are only partially known a rich theory developed in the last decade [15-17] can be used to design robust filters for classical matched or Wiener filtering. These are maximin filters, i.e. they achieve the best possible performance over all spectra in a given class. Usually, one first finds the signal and noise spectral pair which gives the least favorable performance, if it exists. Then one optimizes the filter for the least favorable point, hence the name maximin filter.

For the time delay problem the only published result known to us in this category of processors is that of Kassam and Hussaini [18] where they used the fact that the Eckart processor maximizes a classically defined signal-to-noise ratio to relate the filtering problem to robust hypothesis testing. This is achievable only by associating uncertainty classes with the spectral product $G_{n_1}(\omega)G_{n_2}(\omega)$, rather than with the individual noise spectra themselves. Finding robust solutions to the other signal-to-noise ratios SNR_2, SNR_3, remains an only partially solved problem.

For the pure delay channel an alternate approach to combatting against poor performance with uncertain spectra is suggested by the recent work in robust Wiener filtering [16, 17] when applied to the $W.P.$ With regard to the original formulation of the $W.P.$ we can replace the least mean square estimates of the channel input and the channel output by the robust least mean square estimates of $s(t)$ and $s_0(t)$ under uncertainty in the signal and noises. Specifically we assume that the

signal spectrum $G_{ss}(\omega)$ belongs to the spectral class $\{\sigma\}$, and that the noise spectra $G_{n_1}(\omega)$ and $G_{n_2}(\omega)$ belong to the spectral classes $\{\eta_1\}$ and $\{\eta_2\}$ respectively. Then we solve for the least favorable pairs for Wiener filtering $\{G_{ss}^{LF}(\omega)\,,\,G_{n_1}^{LF}(\omega)\}$ and $\{G_{ss}^{LF}(\omega)\,,\,G_{n_2}^{LF}(\omega)\}$ over the product classes $\{\sigma\times\eta_1\}$ and $\{\sigma\times\eta_2\}$ which yield the robust Wiener filters H_1^R and H_2^R (see Eqns. (2.3) and (2.4)). Finally we implement these filters in the cross-correlation domain as a G.C.C., a scheme which we will call the *Robust Wiener Processor* or the *R.W.P.*

V. NUMERICAL COMPARISONS

At the present time no simulation results concerning the experimental performance of the $W.P.$ and $R.W.P.$ as opposed to the other G.C.C.'s are available. In their absence a preliminary investigation of the relative merits of the above processors was performed based on various signal-to-noise ratio criteria for some specific observation spectra and for the pure delay channel.

Fig. IV - VIII show the relative performance of the $H.T.$, $H.B.$, $Eckart$, $SCOT$, and $C.C.$ under the criteria SNR_1, SNR_2, SNR_3 and local variance of the time delay estimate [3], for a third order Markov signal in first order Markov noises with the noise 3dB bandwidth a factor of ten greater than that of the signal. The interesting thing to note is that under SNR_1 and SNR_2 the $W.P.$ exhibits better performance than all of the other sub-optimum G.C.C.'s for that particular definition of SNR and under SNR_3 is in for a close second next to the $M.L.E.$ In fact, under the criterion SNR_1 performance of the $W.P.$ is virtually identical to the the optimal $H.T.$ processor. The local variance, although it ranks the $W.P.$ behind the $H.T.$, $H.B.$, and $Eckart$ processors, only marginally disfavors the $W.P.$ at low signal-to-noise ratios. (It is to be noted from Fig. VIII that the $SCOT$ and the $C.C.$ have local variance orders of magnitude worse than the $W.P.$).

In Figs. IX and X the performance of the $R.W.P.$, $W.P.$ and other G.C.C.'s are compared using SNR_1 for the ε-contaminated uncertainty class on the specific spectra in the example outlined in Kassam and Lim's paper on Robust Wiener filtering [16]. Specifically, under the nominal

assumption at each sensor we have a signal with the flat bandlimited spectrum $G_{ss}^{o}(\omega)$ in first order Markov noise with the spectrum $G_n^{o}(\omega)$, where the signal and noises are of comparable bandwidths. The uncertainty on the signal and noise spectra are modeled as the ε-mixtures

$$(1-\varepsilon_1)G_{ss}^{o}(\omega) + \varepsilon_1 G_{ss}'(\omega)$$

and

$$(1-\varepsilon_2)G_n^{o}(\omega) + \varepsilon_2 G_n'(\omega)$$

respectively with $G_{ss}'(\omega)$ and $G_n'(\omega)$ arbitrary spectra having the same mass as the nominal and ε_1 and ε_2 lying in the interval [0,1]. Fig. IX shows the relative performance for the nominal spectra and Fig. X the performance for the least favorable signal and noise spectra for Wiener filtering when $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 0.1$. Looking at the nominal case (Fig. IX), we note that the use of the *R.W.P.* entails a loss of about 3dB at low SNR (below about 0db) over the optimal for the least favorable pair. However when the true signal and noise spectra are least favorable for Wiener filtering the *R.W.P.* displays uniformly better relative performance gaining about 3db over the other processors at low signal-to-noise ratios. Note that this is not necessarily the least favorable pair for *H.T.* filtering so that no conclusive result is indicated here. However, Fig. X does suggest that at least for some spectra in the above uncertainty class we can expect better performance with the *R.W.P.* than with the optimal scheme for the nominal spectra.

VI. CONCLUSION

We have outlined the development of several of the most popular G.C.C. implementations and have introduced another G.C.C. which is believed to be new. This G.C.C. should be easier to implement than all but the simple cross-correlator, because the frequency weighting can be estimated directly for unknown spectra. Classical robustness theory led to a simple alteration of the *W.P.*, which we called the *R.W.P.*. The evaluation of theoretical performance in several specific signal and noise environments was presented which suggests that the *W.P.* and *R.W.P.* may be viable alternatives to existing time delay estimation schemes. Further experimental and simulation based performance evaluation is required before any general conclusions can be drawn.

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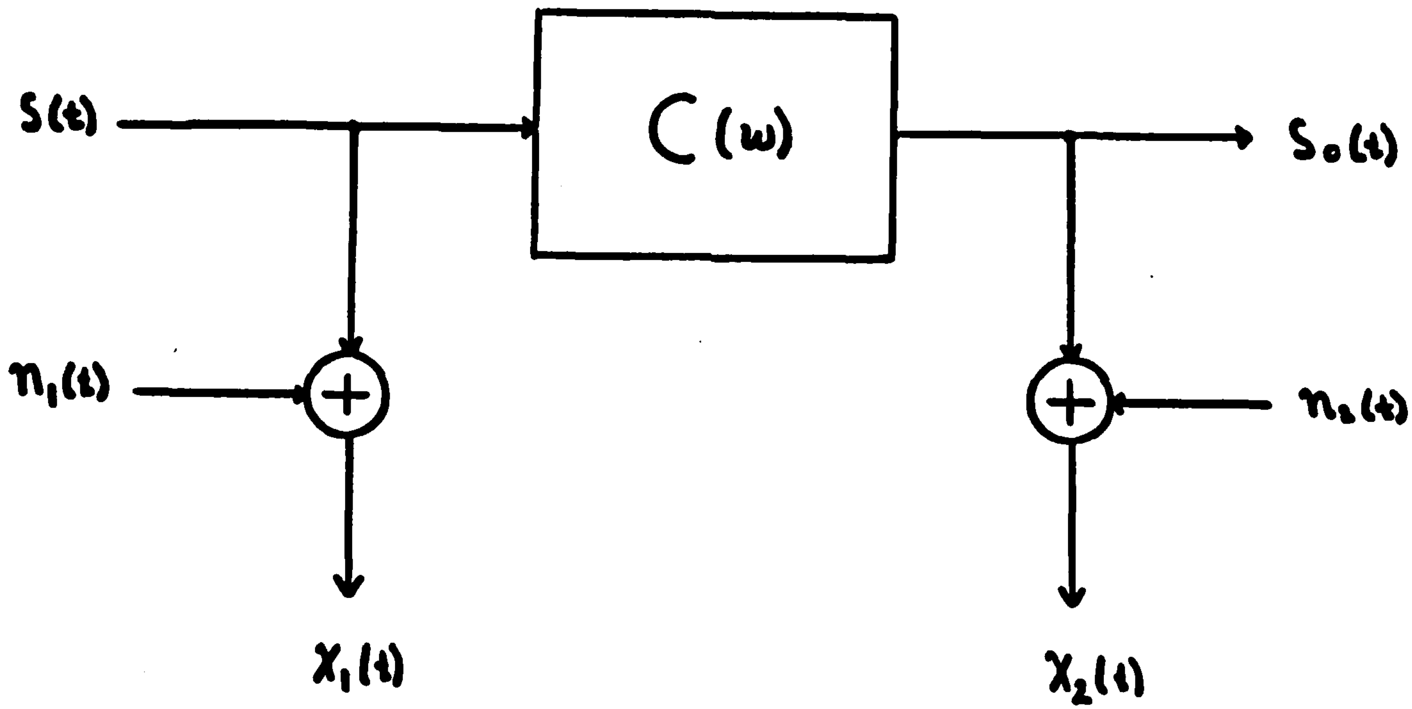


FIG. I

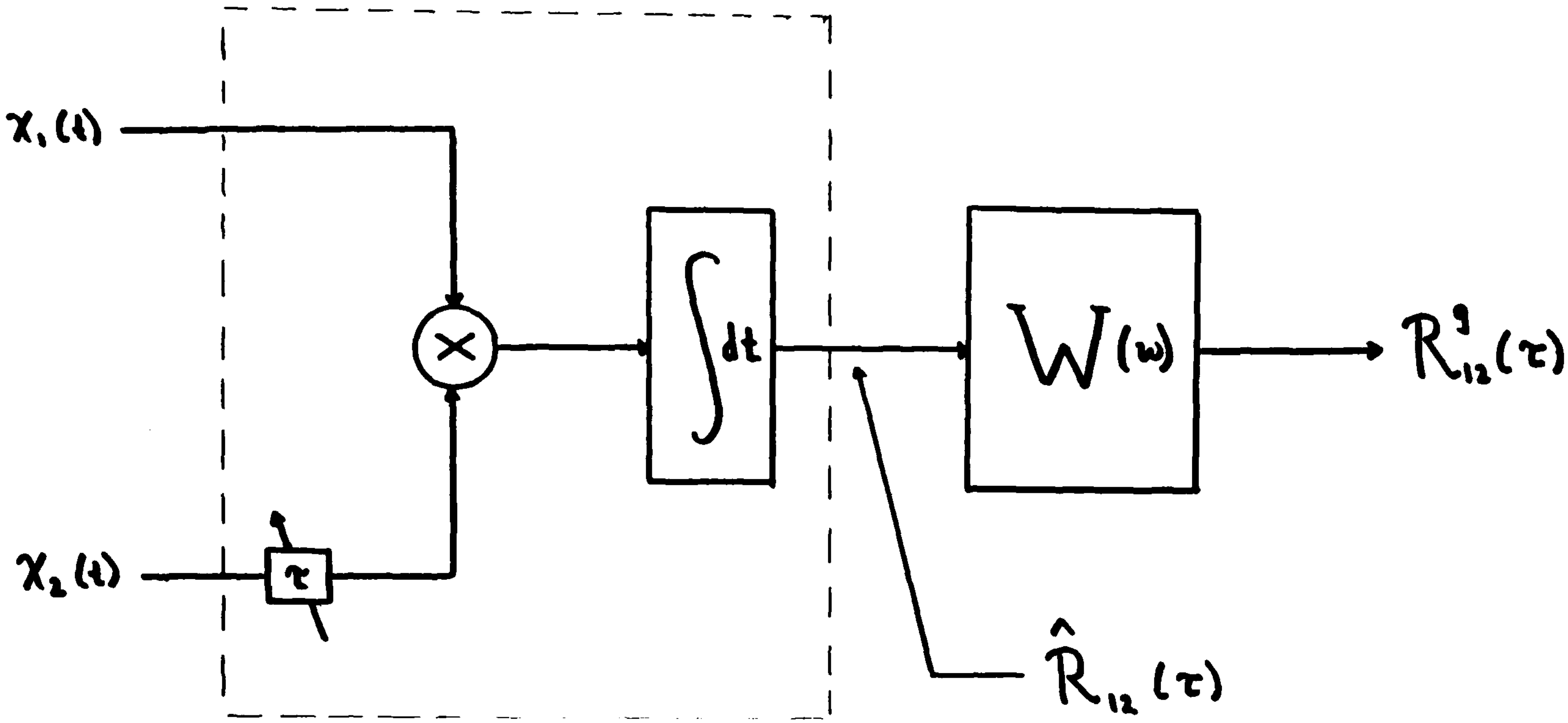


FIG. II

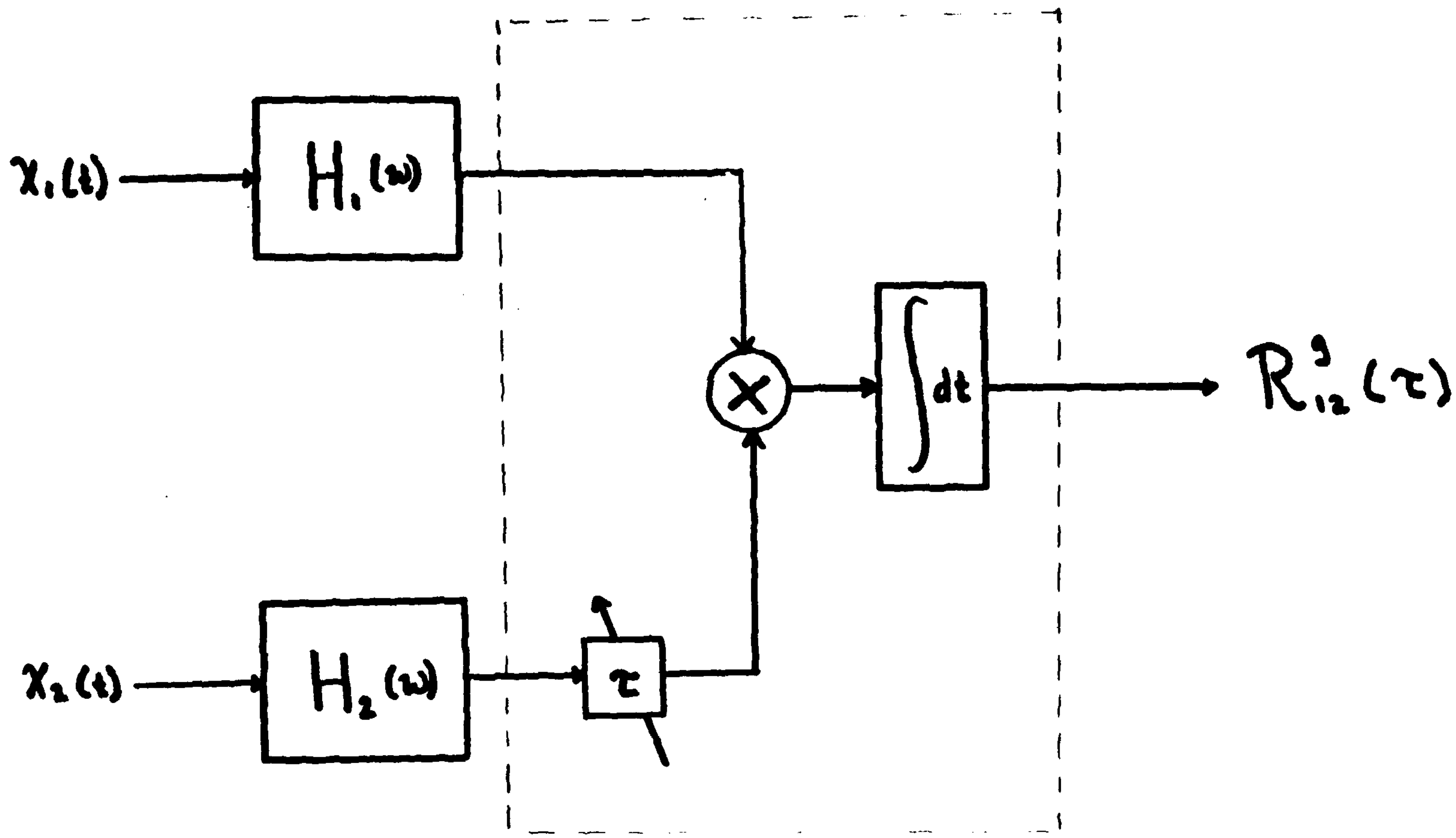


FIG. III

Output SNR1 for Various Processors

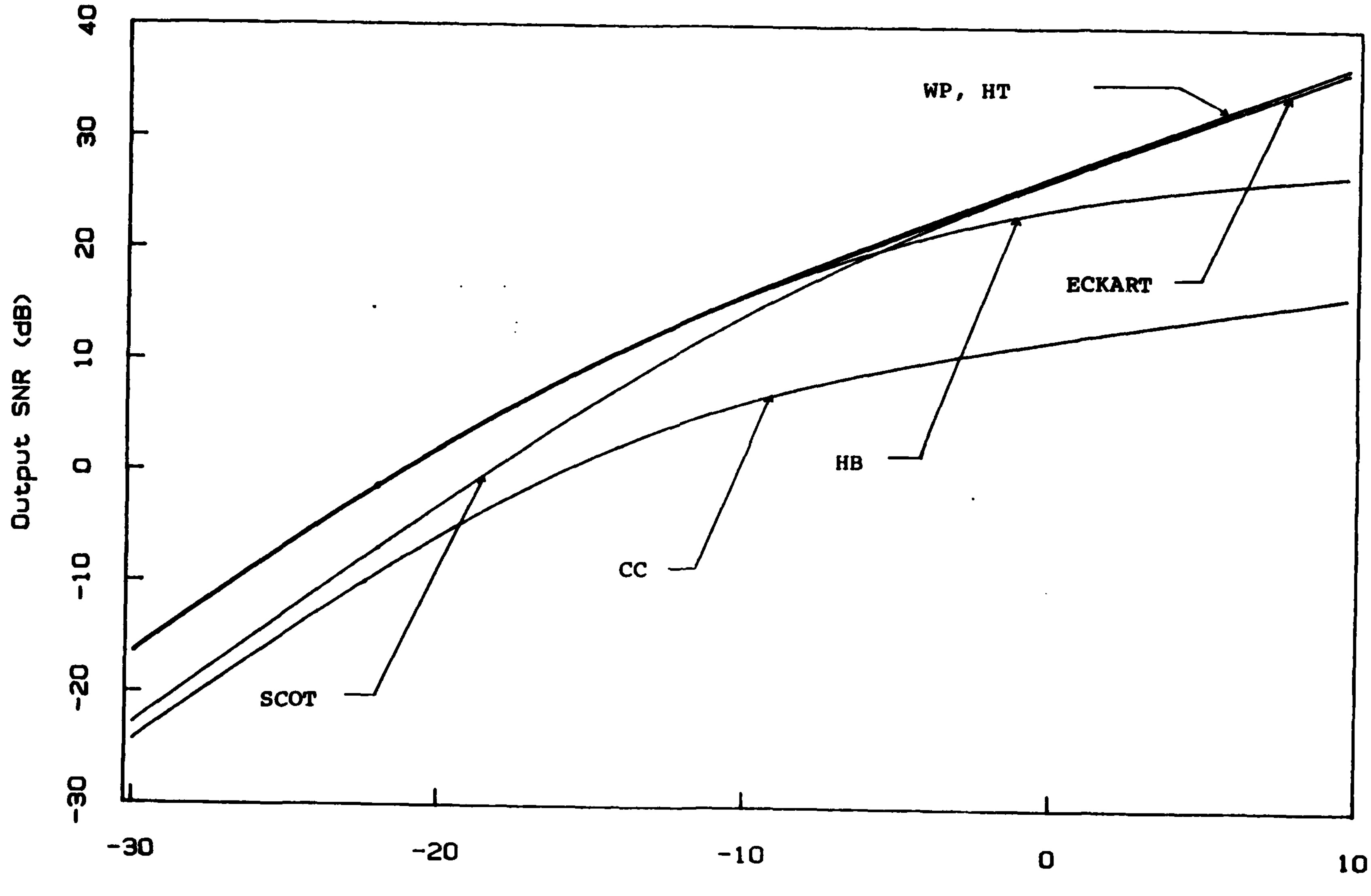
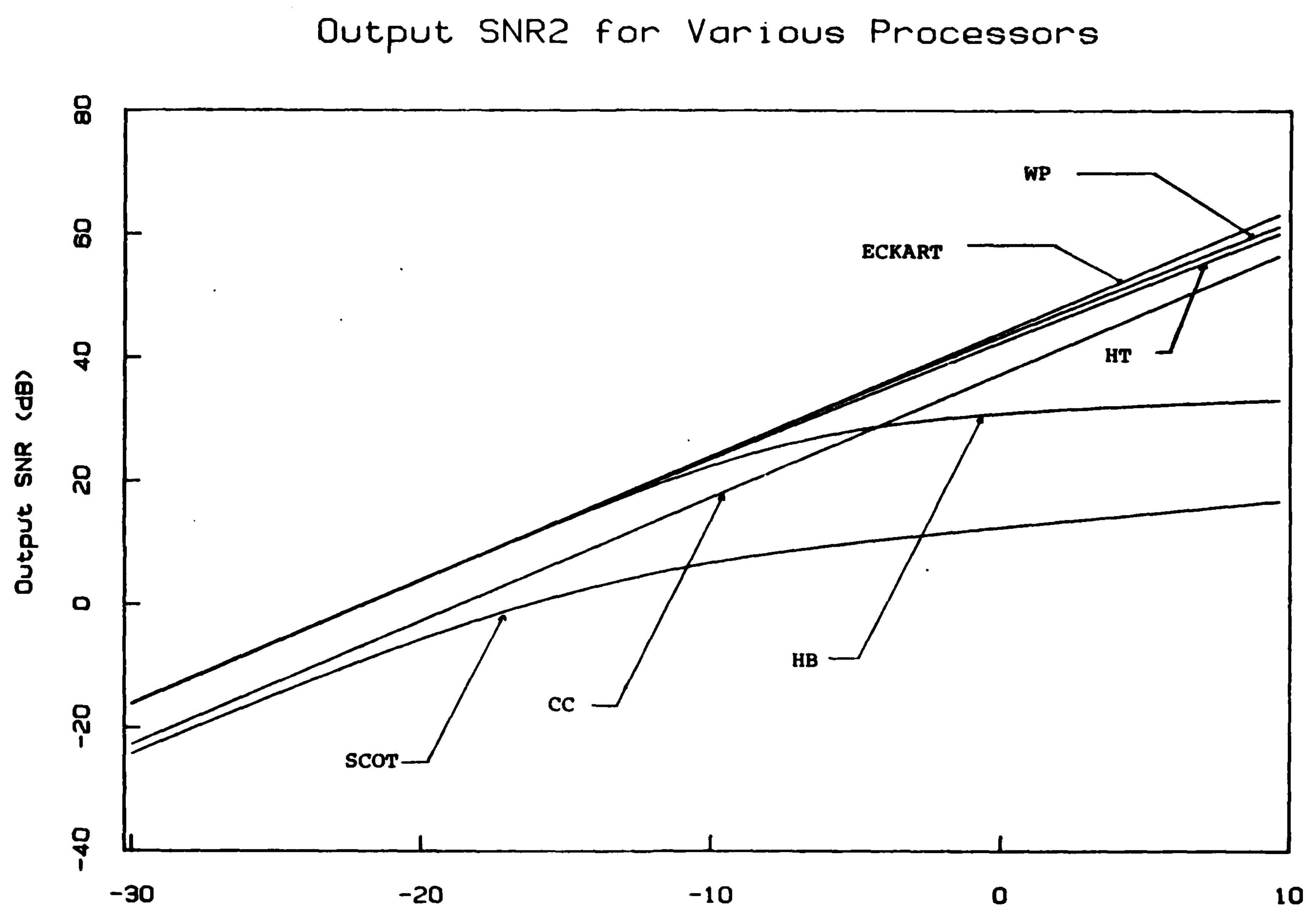
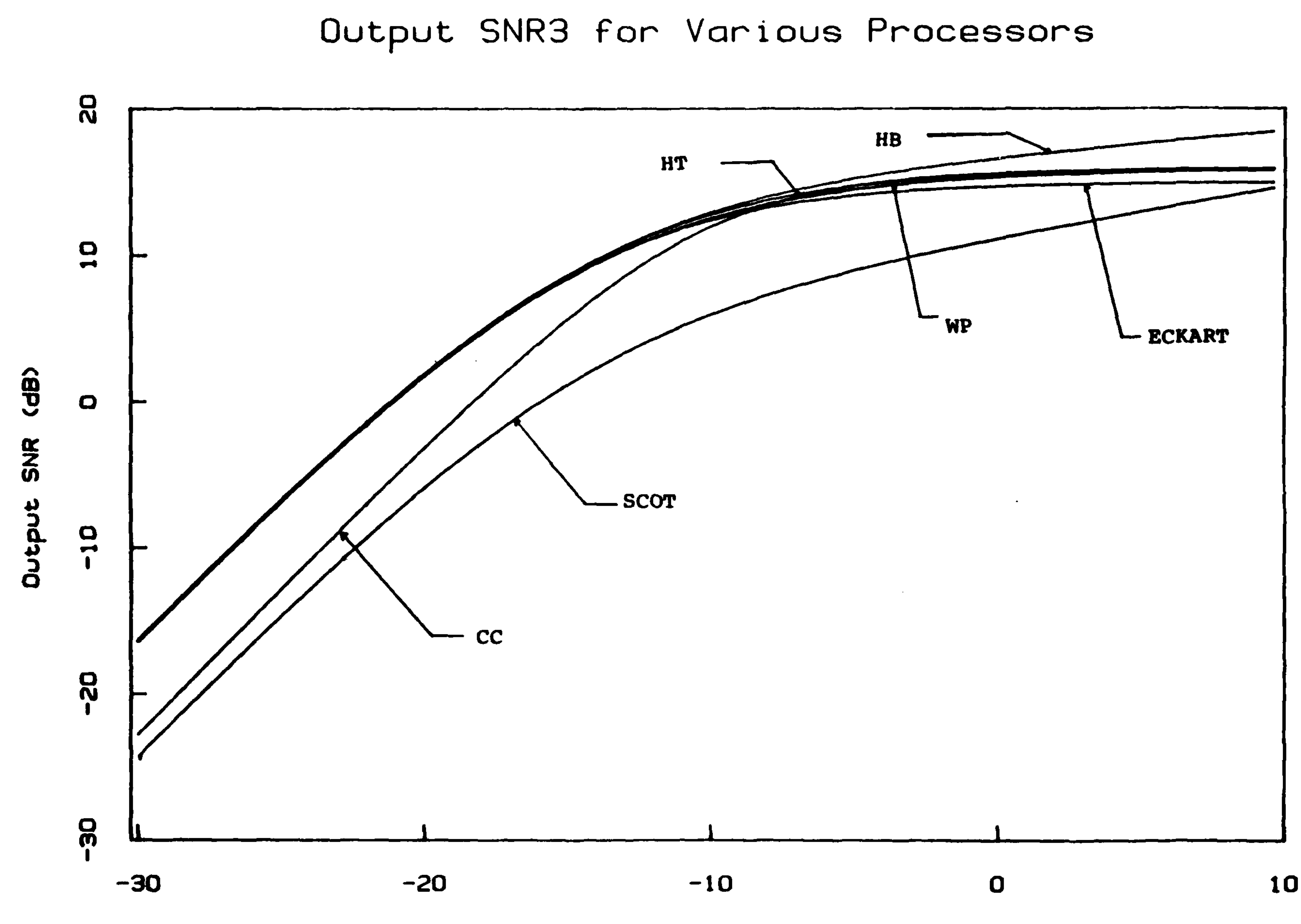


Fig. IV


Output SNR2 for Various Processors
Output SNR (dB)
80
60
40
20
0
-20
-40
-30
-20
-10
0
10
Input SNR (dB)
WP
ECKART
HT
HB
CC
SCOT
Fig. V


Output SNR3 for Various Processors
Output SNR (dB)
20
10
0
-10
-20
-30
-30
-20
-10
0
10
Input SNR (dB)
HB
HT
WP
ECKART
SCOT
CC
Fig. VI

Local Variance for Various Processors

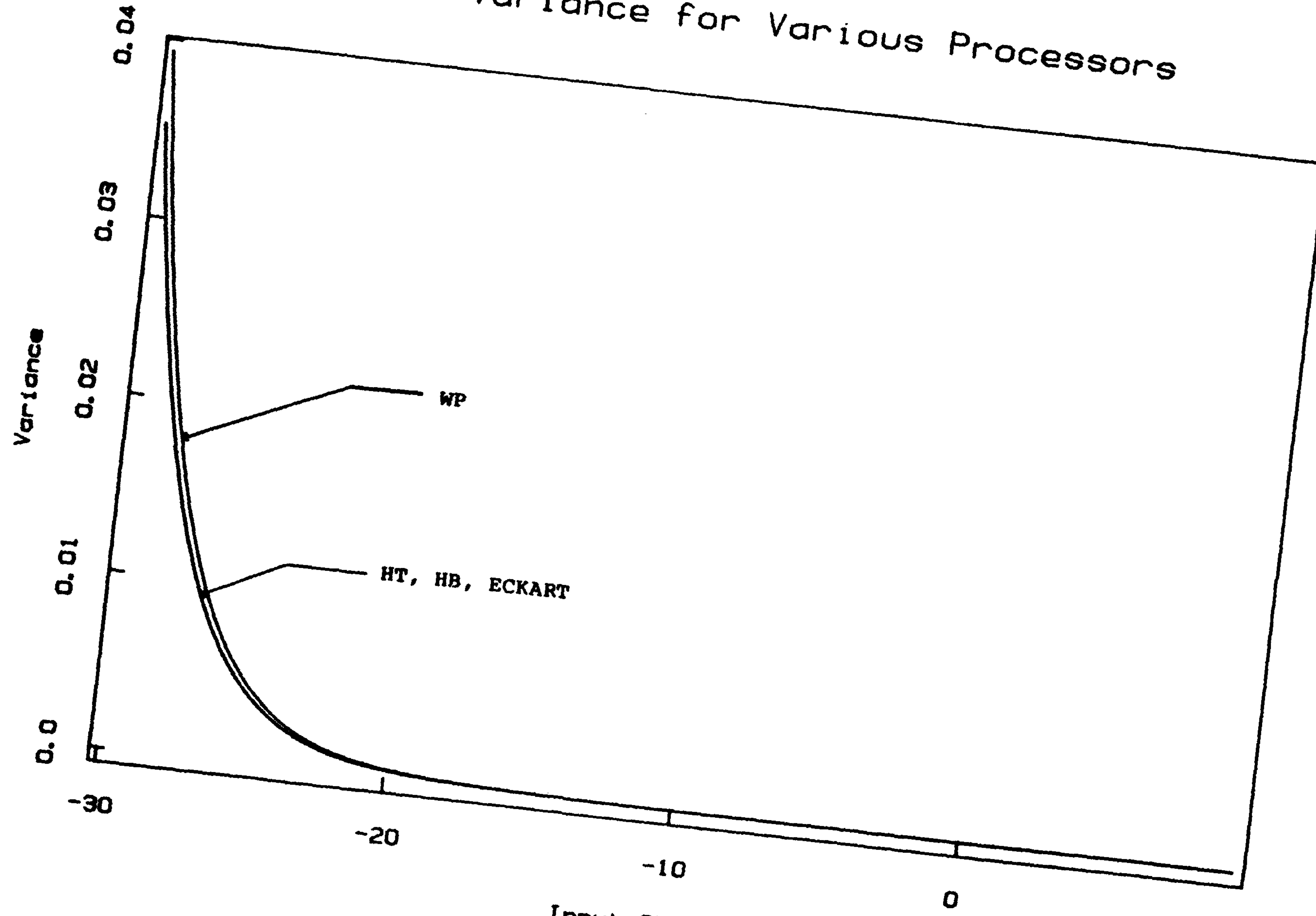


Fig. VII

Local Variance for Various Processors

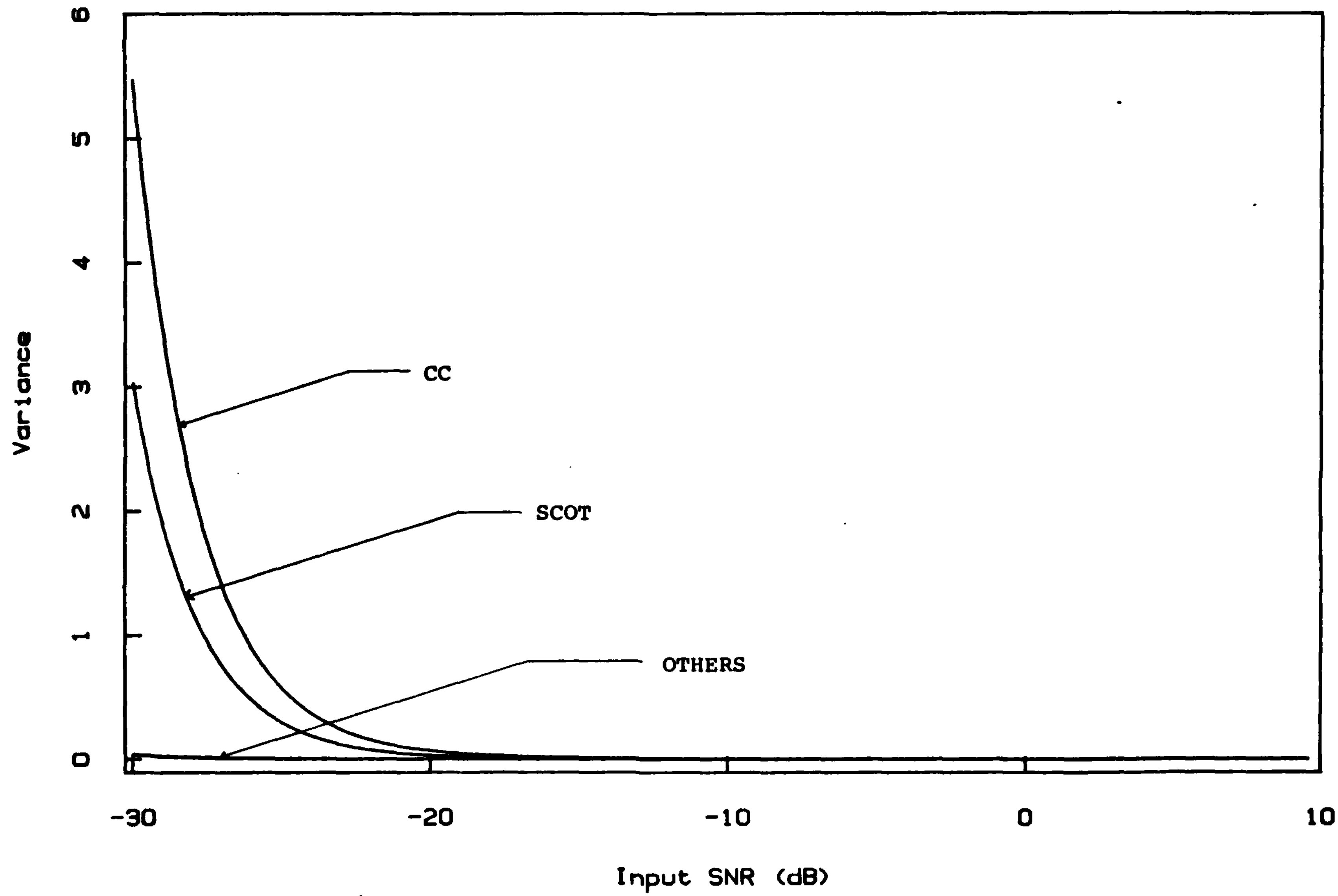


Fig. VIII

Output SNR for Nominal Spectra

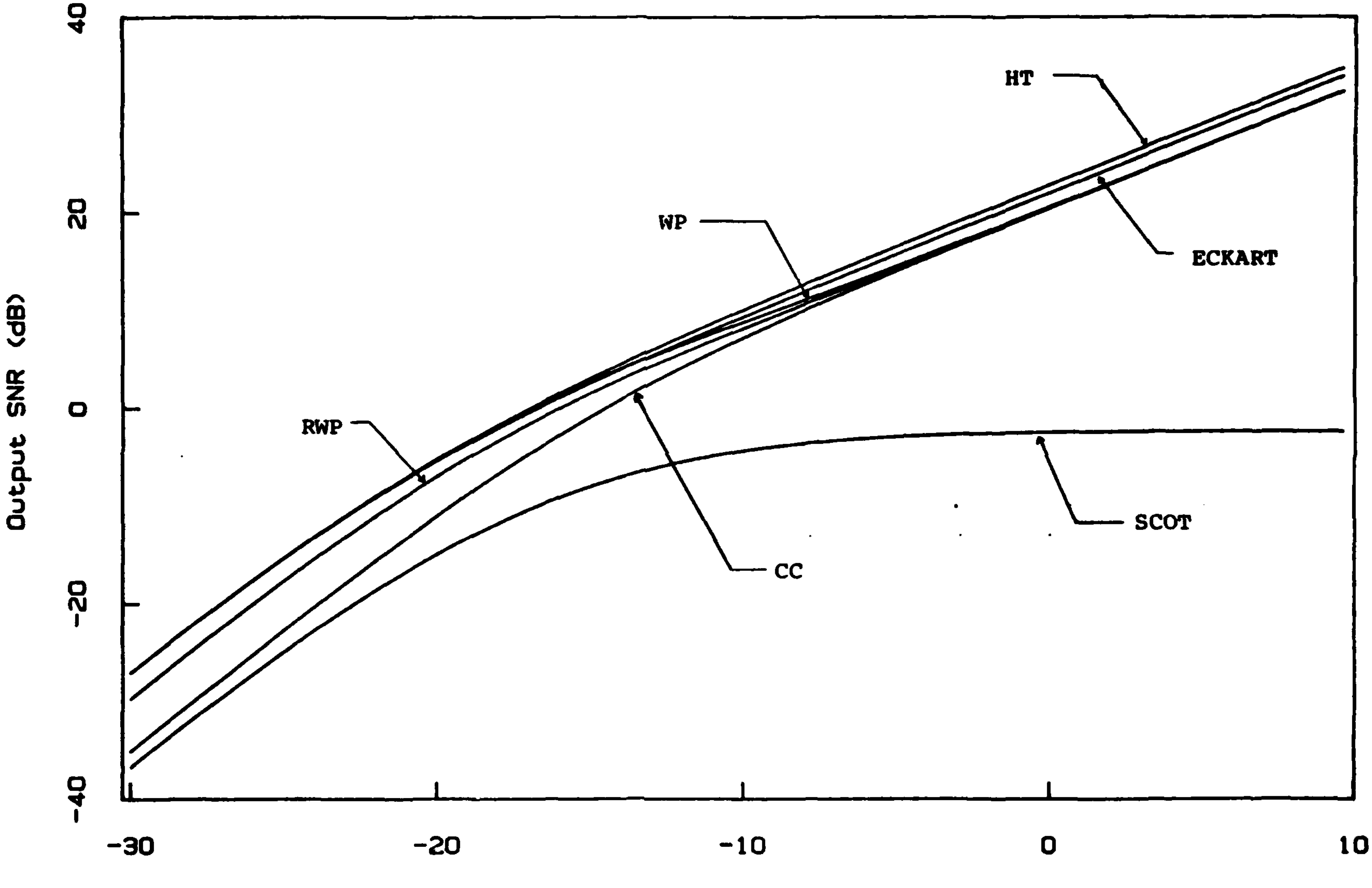
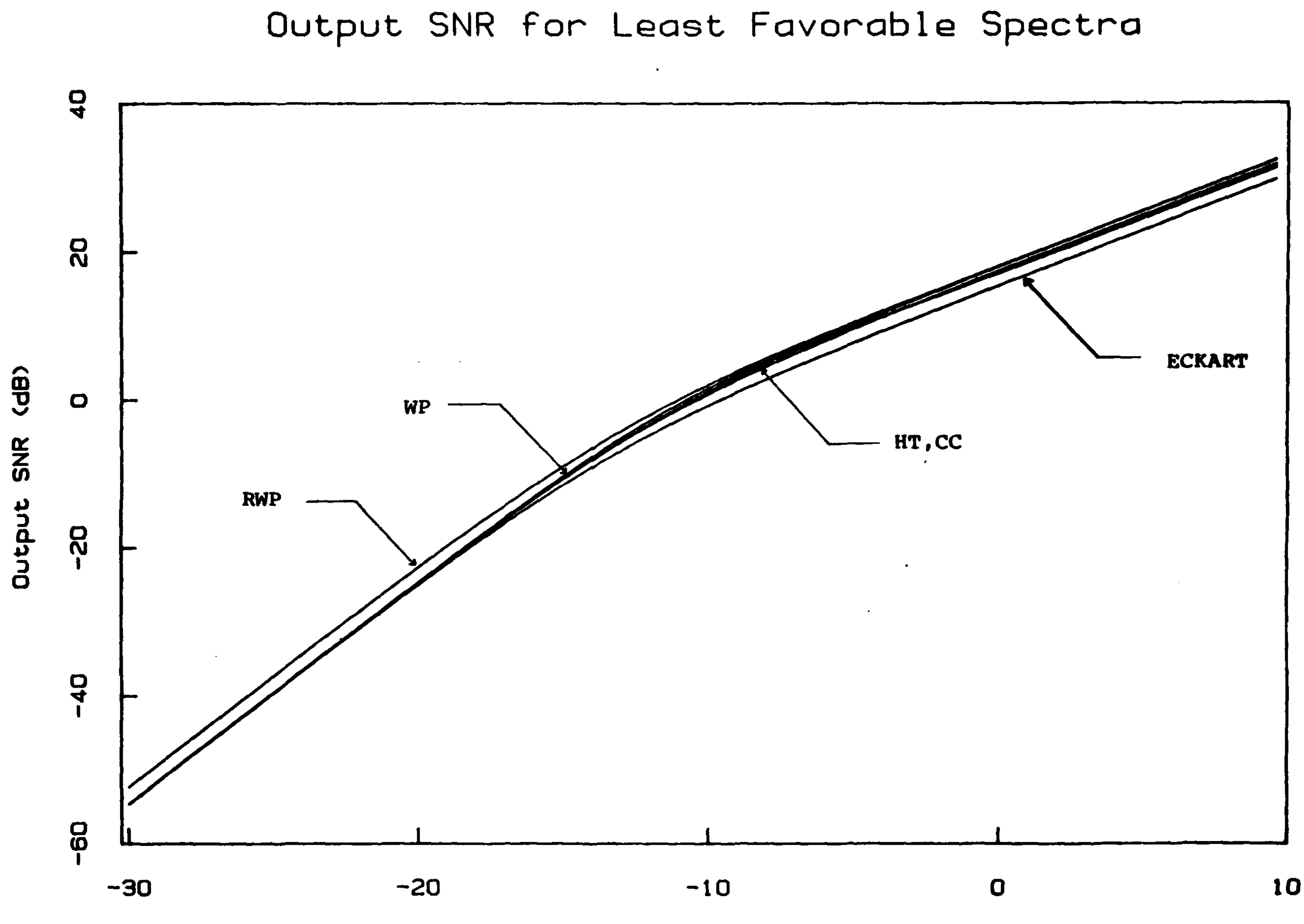


Fig. IX



Output SNR for Least Favorable Spectra
Output SNR (dB)
40
20
0
-20
-40
-60
-30
-20
-10
0
10
Input SNR (dB)
ECKART
HT,CC
WP
RWP

Fig. X

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